A Minimalist Account of Numerals

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ABSTRACT: Numerals participate in the expression of a wide range of operations, including mass, volume, degree, ordering, counting, and arithmetic calculations. This raises the questions of what they denote semantically and how they are derived morpho-syntactically. Although a number of theories have been advanced regarding their semantics, studies on the syntactic side are rather scarce. Further, the syntactic accounts of numerals date back to GB period, calling for a reinterpretation of their conclusions under Minimalist considerations. This study attempts to develop a syntactic account of numerals under Minimalist desiderata. It is proposed that numerals are number-denoting type *n* objects, derived from two primitives: saturated DIGITs of type *n*, and unsaturated BASEs of type <n,n>, instrumental in the derivation of simplex and complex numerals, respectively. This view is demonstrated to account for a wide range of distributional and interpretive possibilities of numerals as well as provide principled reasons for why some plausible forms are consistently unattested across languages.

Keywords: numeral, digit, base, packaging strategy, phrase structure rules

Sayısal İfadelerin Minimalist İncelemesi

ÖZ: Sayısal ifadeler, kütle ve hacim belirtme, derecelendirme, sıralama, sayma ve aritmetik işlem yapma gibi çeşitli süreçlerin ifade edilmesinde etkin bir rol oynamaktadır. Bu durum sayısalların dilbilimsel doğasına, özelikle de anlambilimsel olarak ne ifade ettiklerine ve biçimbilim-sözdizim modülünde nasıl türetildiklerine ilişkin bir dizi soruyu gündeme getirmektedir. Ancak, sayısalların anlamı üzerine çok sayıda kuram öne sürülmüş olsa da sözdizim tarafındaki çalışmalar nitelik ve nicelik bakımından oldukça yetersizdir. Bunun yanında, sözdizim alanında yapılan çalışmaların çoğu Yönetim ve Bağlama döneminden kalma olup bu dönemde varılan sonuçların günümüz çalışmalarına yön veren Minimalizm çerçevesinde yeniden ele alınması gerekmektedir.

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Sayısalların sözdizimsel yapısını Minimalist çerçevede yeniden ele almayı amaçlayan bu çalışmada, bunların sayı bildiren *n* tipinde varlıklar olup iki farklı özden türetildiği öne sürülmektedir: *n* tipinde doymuş RAKAMLAR ile <n,n> tipinde doymamış TABANLAR. Bunlardan ilki basit, ikincisi ise karmaşık sayısalların türetiminde rol almaktadır. Bu yaklaşım ile, hem sayısalların dağılım ve yorumunu etkileyen kısıtlamaların hem de olası bazı türetimlerin dünya dillerinde şimdiye dek gözlemlenememe nedenlerinin ilkeler bazında açıklığa kavuşturulabileceği gösterilmektedir.

Anahtar sözcükler: sayısal, rakam, taban, paketleme stratejisi, öbek yapısı kuralları

1 Introduction

Numeral constructions have raised a lot of interest in linguistics studies, with several theoretical questions being in focus. Numerals are epitomic examples of the generative power of human thought and human language. With so few primitives, the human mind can generate a truly infinite set of numerals. Conceptually speaking, numerals perform a wide range of functions. They are used in expressions of cardinality (1a), measurement (1b), degree (1c) age (1d), order (1e), mathematical calculations (1f), and counting (1g), to name a few.

(1) Contexts involving numerals
  a. four apples
  b. four kilos/liters
  c. four degrees
  d. four years old
  e. the fourth
  f. 4+3=7
  g. 1, 2, 3, …, ∞

Given that numerals are instrumental in expressing these conceptually different functions, a question arises as to what a numeral semantically denotes and how it participates in the expression of these functions.

This makes the syntax of numerals all the more important under the assumption that semantics is read off the syntactic structure. Any lay person's intuition is that numerals come in two variants: simplex ones like *four*, and complex ones like *three hundred and forty-eight*. The literature has more or less converged on the conclusion that numerals are phrasal objects derived by a generative mechanism, as evidenced by the presence of the coordinator *and* in the derivation of complex numerals (Ionin & Matushansky, 2006).

Further, not all plausible combinations produced by this generative mechanism are attested crosslinguistically, leading to the postulation of some constraints that regulate the way chunks of complex numerals are combined. The most influential of such constraints is Hurford's (1975; 1987; 2007) Packaging Strategy, which roughly states that chunks denoting higher-valued numerals are ordered to the left of lower-valued ones. Proposed at a time when the working of language was explained by phrase structure rules, Packaging Strategy has nevertheless been assumed more or less as is in later analyses. With Minimalism, however, linguistic theorizing has moved away from phrase structure rules, eventually adopting a mode of derivation in which all syntactic operations are motivated. In this model, the notions of economy and simplicity play a significant role.

This study attempts to fill this gap: analyze the syntactic structure and the resulting semantic interpretation of numerals under Minimalist desiderata. Three issues will be discussed in particular: (i) what type of entities numerals are that allows them to appear in the wide range of contexts in (1), (ii) what their internal syntactic structure looks like, and (iii) why certain combinations are largely unattested. Building on the success of earlier works, I develop an account of cardinal numerals that is simpler and more economical substantively as well as derivationally. In doing so, I will also assume a revised version of Hurford's (1975; 1987; 2007) Packaging Strategy, and demonstrate that some unattested derivations are ruled out in principle.

The organization of the paper goes as follows. Section 2 briefly reviews Hurford's (1975; 1987) and Booij's (2009) analyses, and highlights some areas in need of improvement in accordance with Minimalist considerations. The analysis itself is developed in Section 3 where I first review four different accounts on the semantics of numerals, and eventually adopt the Platonistic view that numeral phrases (NumPs) are *n*-type objects that only denote a natural number. I then propose a bottom-up derivation according to which NumPs are derived from two basic primitives: DIGITs of type *n*, and BASEs of type <n,n>. Multiplicative numeral-formation processes are shown to be instances of saturation, driven by the type requirements. Revising Packaging Strategy in accordance with Minimalist requirements, I then demonstrate how the proposed model filters out unattested cases with no additional stipulations. This section finishes off with a discussion on the status of plausible but unattested BASEs. Section 4 addresses some non-standard cases and what implication they have for the presented model. More specifically, I discuss subtractive numeral-formation processes, and the divisive nature of fractions, all pointing to the conclusion that numerals can be derived by division as well as subtraction alongside the most commonly exploited mechanism of addition and multiplication.

2 Earlier Analyses

This section reviews Hurford's (1975; 1987) and Booij's (2009) accounts of complex numerals, and highlights the areas in need of improvement so as to fit Minimalist considerations.

2.1 Hurford (1975; 1987)

Hurford (1975; 1987) proposes that numerals are derived from two basic primitives: DIGITs and Ms. The former consists of simple numerals like *one, two, three, …, nine*, and the latter consists of multiplicative bases like *-ty* (for ten), *hundred, thousand, million*, … The generative system that derives numerals employs two Phrase Structure Rules (PSRs) that operate on these primitives.

(2) Universal Phrase Structure Rules

a. $\text{NUMBER} \rightarrow \left\{ \begin{array}{c} \text{DIGIT} \\ \text{PHRASE (NUMBER)} \end{array} \right\}$ (interpreted by addition)

b. $\text{PHRASE} \rightarrow \text{(NUMBER) M}$ (interpreted by multiplication)

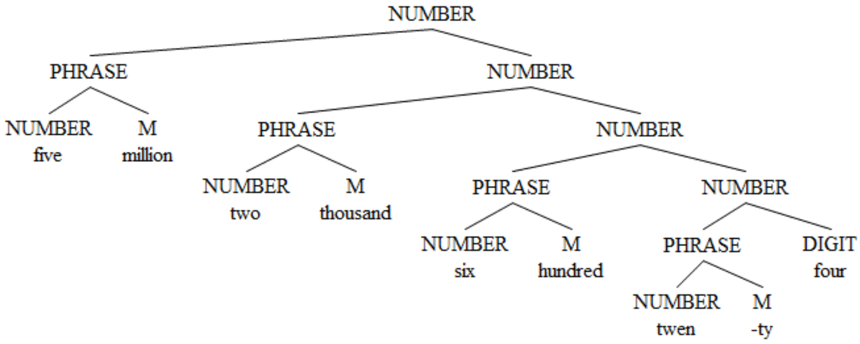
Here, curly braces indicate options among which the system can select, and parentheses indicate optionality. A numeral (i.e. NUMBER) may consist of a DIGIT only, or a PHRASE, which may optionally combine with another NUMBER. PHRASE, on the other hand, may consist of an M only, or a NUMBER and an M. In this view, the derivation of simplex and complex numerals in (3) goes as follows.

(3)

	Numeral	Rule	Computation	Output
a.	4	DIGIT	4	*four*
b.	400	PHRASE: NUMBER M	4x100	*four hundred*
c.	468	[PHRASE: NUMBER M] + [PHRASE: NUMBER M] + DIGIT	4x100+6x10+8	*four hundred sixty-eight*

The system allows recursivity on the category NUMBER since NUMBER may go to PHRASE (NUMBER), and PHRASE may go to (NUMBER) M, all the way to infinity. Structurally speaking, the representation of *5,002,624* is as in (4).

(4) *5,002,624*



Hurford is aware that, left as is, the system massively over-generates. Thus, although the PSRs produce all the combinations in (5), only (5a) is a legitimate Spellout for *345*.

(5) *345*

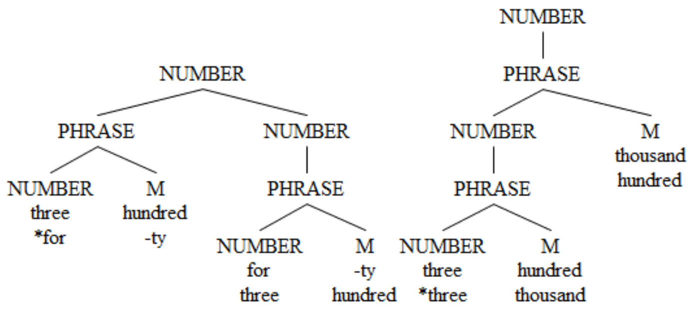
a. three hundred forty-five b. *forty-five three hundred
c. *five three hundred forty d. *forty three hundred five

To eliminate the unattested cases, Hurford proposes that the output of the PSRs in (2) is subject to the Packaging Strategy (PS), a well-formedness constraint on the ordering of NUMBERs relative to one another.

(6) Packaging Strategy
The sister constituent of a NUMBER must have the highest possible value.
(Hurford, 1975, p. 67)

By PS, higher-valued constituents are located higher in the tree. This ensures that they appear to the left in additive constructions, and to the right in multiplicative ones. In a sense, the system works "cyclically from the top of a tree downwards, to get the highest numerals all the way to the top" (Hurford, 2007, p. 775).

(7) a. Additive b. Multiplicative
340 *300,000*



In (7a), *three hundred* occurs to the left of *forty*, with which it combines additively. In (7b), *thousand* occurs to the right of *three hundred* with which it combines multiplicatively. Crucially, though, the higher-valued *three hundred* in the former and *thousand* in the latter occur structurally higher than the lower-valued *forty* and *three hundred* respectively. Note incidentally that, since the notions of Spec-Head-Complement were missing at the time, the question of how directionality follows from the configurations in (7) did not come up in the first place.

In a more recent work, Hurford (2007) proposes that PS is the result of two general and culturally evolved principles.

(8) a. Go as far as you can with the resources you have.
 b. Minimize the entities you are dealing with.

(8a) is intended to account for the existence and use of Ms, and explained in Hurford (2007) by analogy to the practice of carrying apples. When carrying apples in baskets, the argument goes, rather than dividing them into equal amounts for each basket, we tend to fill our available baskets up to the top with apples, and leave the remaining apples for a final basket, which may or may not be full. A similar practice is argued to hold for numerals: When producing, say, *34*, we first fill three "baskets" (the Ms of *ten*s in this case) up to the top, and then reserve the remaining *4* for another basket. This explains the grammaticality contrast between (9a) and (9b), both involving addition.

(9) *14*
 a. four-teen
 b. *seven (and) seven

In (9a), *14* is spelled out as *four-teen*, (i.e. 1x10+4, with subsequent reordering of the morphemes *ten* and *four*). In (9b), however, the baskets of *ten* are filled equally as *seven (and) seven*, in violation of the (8a), i.e. *Fill in the Ms as fully as possible*.

(8b), on the other hand, is brought in to explain the grammaticality contrast in (10).

(10) *3000*
  a. three thousand
  b. *thirty hundred

Hurford (2007) argues that when we reach *1,000* by counting the Ms of *hundreds*, we have an apparent choice between going on to count in *hundreds* and putting the *ten hundreds* we now have into a single package called *thousand*. By following the second mechanism, we are essentially reducing the number of packages in our resulting numeral from "thirty" to "three", hence the term *Minimize entities (i.e. the number of Ms) you are dealing with*.

Although Hurford's model is highly successful in deriving complex numerals as well as filtering out unattested cases, it does not quite fit with the Minimalist framework for several reasons. First, Hurford bases his analysis on a set of Phrase Structure Rules (PSRs), which have gone out of favor in linguistic theorizing. Second, Minimalism favors motivation-based analyses over purely technical accounts like PSRs. A linguistic derivation is taken to start by copying a set of lexical items from the lexicon (called a numeration) and proceed by combining them through the operation Merge, which is sensitive to the requirements (i.e. features) of the lexical items (Chomsky, 2000; 2001). As such, there must be a reason for why and how an M combines with a NUMBER, optionally or obligatorily. Third, with Extension Condition (Chomsky, 1993; 1995), derivation is assumed to work bottom-up, with successive applications of Merge at the root of the tree. As such, Hurford's proposal that the system combines NUMERALS in a top-down fashion can no longer be maintained.[1] Fourth, there is nothing intrinsic in Hurford's model to prevent *four hundred thirty* as a possible Spellout for (7a). How does the system know that *hundred* should compose with *three* and *four* should compose with *-ty* but not vice versa? In Minimalism, such undesired outcomes are prevented by assuming that the numeration is structured into subarrays (Chomsky, 2000). Since the notions of numeration and subarray are missing in Hurford's model, the question did not come up in the first place. Fifth, the principles in (8), despite being essentially correct, do not make much sense to a linguist unless expressed in Minimalist terms. Finally, Hurford's account

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[1] Hurford (2007, p. 775) himself notes that "a top-down algorithm is not absolutely necessary to implement the strategy".

says nothing regarding what type of entities numerals are, and what sort of interpretation obtains from their semantic composition. This is a non-trivial issue given that numerals play a crucial role in expressing the wide range of functions in (1).

Thus, Hurford's account of numerals needs some revising so as to handle the same set of data under a Minimalist perspective. We minimally need to (i) restate the principles Hurford proposes in Minimalist terminology, (ii) modify PS to handle bottom-up derivation, (iii) motivate his PSRs, (iv) derive additive numerals by coordination, and finally (v) associate the emerging syntax with an appropriate and uniform semantic interpretation.

2.2 Booij (2009)

Building his analysis on Hurford (1975; 1987; 2007), Booij proposes a feature set that captures the distribution of different lexical elements used in the derivation of Dutch numerals. Different numeral expressions are specified with binary values of three features: [±Num], [±M], and [±N].

(11) Features of numeral expressions in Dutch
  a. [+Num, -M, -N] *een* 'one', *twee* 'two', *drie* 'three', …
  b. [+Num, +M, +N] *honderd* 'hundred', *duizend* 'thousand', …
  c. [-Num, +M, +N] *miljoen* 'million', *miljard* 'billion', …
(adapted from Booij (2009, p. 10))

The feature [Num] describes whether an item can denote a numeral on its own: [+Num] can, [-Num] cannot (12a). [N] describes whether the item can be pluralized like nouns: [+N] can, [-N] cannot (12b). Finally, [M] specifies whether an item can appear in the singular form after a numeral: [+M] can, [-M] cannot (12c).[2]

(12) a. {*drie* / *honderd* / **miljoen*} boek-en
  three hundred million book-PL
  '{three / hundred / *million} books'
 b. {**deri-en* / *honderd-en* / *miljoen-en*} boek-en
  three-PL hundred-PL million-PL book-PL
  '{*threes / hundreds / millions} of books'

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[2] Booij (2009) follows Hurford (1975) in taking [+M] numerals like *hundred* and *thousand* to be measure words, hence [±M].

c. twee {**drei / honderd / miljoen*} boek-en
two three hundred million book-PL
'two {*three / hundred / million} books'
(adapted from Booij (2009, p. 10))

It is important to note at this point that Booij develops his account within the framework of Construction Grammar, according to which constructions exist as lexical primitives on a par with words and morphemes that provide sound-meaning pairings. Booij gives the caused motion construction in (13) as an example.

(13) Pat sneezed the foam off the cappuccino.

Here, a typical intransitive verb like *sneeze* is used transitively, which poses a challenge given that the object *the foam* cannot receive a theta role from it. This is possible, Booij argues, because the construction is listed as such in the lexicon, with a specification as to what sort of interpretation it will receive. In a sense, constructions are like idioms that receive a holistic rather than a compositional interpretation (Jackendoff, 2008; 2011).

In accordance with Construction Grammar, Booij goes on to provide schemas that derive the multiplicative interpretation obtained by combining DIGITs and Ms (14a) and the additive interpretation for numerals greater than 100 (14b).

(14) a. Multiplication schema
$[\mathrm{Num}^i\ \ \mathrm{Num}^j_{[+M]}\ ]^k_{\mathrm{Num}} \leftrightarrow [\mathrm{NUM}^i \text{ x } \mathrm{NUM}^j]^k$
b. Addition schema for numerals > 100
$[\mathrm{Num}_C{}^*\ ((\varepsilon n)\ \mathrm{Num}_D)]^j_{\mathrm{Num}} \leftrightarrow [\mathrm{NUM}_C + \mathrm{NUM}_D \ldots]^j$
(Booij, 2009, pp. 10-11)

Details aside, the part before ↔ represents the form of the lexically listed schema, and the part after ↔ represents the interpretation it receives.

Note crucially that the Minimalist tradition rejects the relevance of constructions/schemas as possible lexical primitives, and argues instead that, by the Headedness Principle, all phrases must be the projection of a head. As such, Booij's proposal cannot be maintained unless restated in Minimalist terms. In particular, the schemas need to be expressed as syntactic structures, and the interpretive rules as natural outcomes of semantic composition. Further, the mere postulation of feature specification, descriptively correct as it may be, is not particularly helpful in understanding the phenomena. Why should some numeral expressions like *hundred* be specified as [+Num] and others like *million* as [-Num]? Could the difference be connected to some more deeply rooted semantic facts? Ideally, idiosyncratic phenomena like lexical specification should not be brought into picture unless other means are exhausted, and I demonstrate in Section 3 that the same distributional difference can be captured without reference to features.

This study demonstrates that the set of facts intended to be captured under Hurford's (1975; 1987; 2007) and Booij's (2009) models can be accounted for through Minimalist principles. Better yet, the derivation of numerals in such an account not only exploits independently established constraints, but also proves to be much simpler and economical both substantively and derivationally. The next section addresses this issue.

3 Derivation of Numerals

3.1 Semantics of Numerals

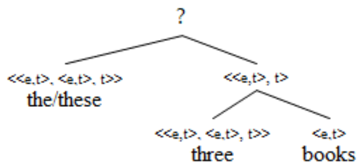
We should start by asking what sort of entities numerals are and work our way backwards to how this interpretation is achieved compositionally in the case of complex numerals. In doing so, we need to consider the bare fact that numerals are obligatorily involved in the expression of the wide range of functions listed in (1): cardinality, measure, degree, counting, calculation, etc. The success of a theory of numeral semantics should be judged by how well it accounts for their role in the totality of these varying functions rather than simply focusing on cardinality expressions.

There are essentially four theories of numerals proposed in the literature, which I review here briefly.

3.1.1 Numerals as determiners of type <<e,t>, <<e,t>, t>>

The first account of numerals, advocated in Montague (1974), Bennett (1974), Barwise and Cooper (1981) and van der Does (1993) among others, is that numerals are <<e,t>, <<e,t>, t>>-type determiners. This position, however, cannot be maintained given the wildly different semantics between quantificational determiners and numerals: The former expresses a relation between two sets while the latter are used in expressing such functions as cardinality, measurement, etc. Distributionally speaking, a numeral can, and in fact must, participate in the derivation of measure phrases while a determiner cannot (15a). Moreover, a determiner and a numeral can cooccur (15b), which leads Ionin and Matushansky (2006) to conclude that their combination does not yield an interpretable derivation.

(15) a. {three / *the/??these} kilos
b. the/these three books



Further, determiners are syntactic heads while numerals are phrases. Thus, both interpretive and distributional factors strongly disfavor an account of numerals assigning them to the category and type of determiners.

3.1.2 Numerals as predicates of type <e,t>

Another theory maintains that numerals are <e,t>-type predicates (Partee, 1987; Link, 1987; Verkuyl, 1993; Carpenter, 1998; Landman, 2003). This, too, is quite problematic. For one thing, predicates denote a property, and thus can be used in attributive as well as predicative positions. However, although both numerals and <e,t>-type adjectives can surface in attributive positions, predicative positions strictly disallow the former (16a). For another, while most (gradable) adjectives accept degree modifiers, numerals strictly disallow them (16b). On the other hand, only numerals can participate in the expression of measures (16c).

(16) a. The waiters are {polite/*three}.
b. quite {nice/*three}
c. {three/*nice} kilos of apples

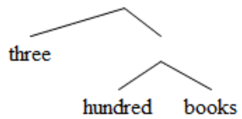
Further, although combining two predicates would not give rise to a type mismatch, Ionin and Matushansky (2006) note that such a composition would yield incorrect truth conditions. This is because two <e,t>-type predicates can only be interpreted as an instance of property intersection. As such, a complex numeral expression like *three hundred books* would necessarily yield the implausible reading that the books are simultaneously *three* and *hundred* in cardinality.

Once again, both interpretive and distributional contrasts argue against treating numerals as predicates of type <e,t>.

3.1.3 Numerals as modifiers of type <<e,t>, <<e,t>>

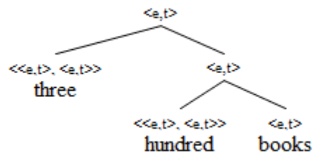
A further analysis of numerals comes from Ionin and Matushansky (2006), who propose that numerals are type-<<e,t>, <e,t>> modifiers. Their account is based on the assumption that complex numerals are built by iteratively taking one another as complements, as in (17).

(17) three hundred books



Here, the system first builds *hundred books*, which is then taken as a complement by *three*. Based on this representation, Ionin and Matushansky (2006) argue that the only way to derive the correct semantics of (17) without giving rise to a type mismatch is by assuming that numerals are modifiers of type <<e,t>, <e,t>> whereby each numeral in a multiplicative construction modifies its complement.

(18) three hundred books



This model does not suffer from the problems of type clash inherent in the numerals-as-determiners analysis, and the incorrect truth conditions that plagued the numerals-as-predicates analysis. It correctly captures the resultant interpretation that books are *hundred* in cardinality, and this *hundred* occurs *three* times[3]. Nevertheless, the modifier account fails to explain other functions of numerals like measuring, counting, calculation, etc.

(19) a. three hundred kilos of flour (measuring)[4]
b. three times four equals twelve (calculation)

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[3] Still though, this argument is questionable, given that *three* and *hundred* stick together under extraposition.

(i) a. John bought books, *three hundred* in number.
b. *John bought *hundred* books, *three* in number.

[4] A reviewer notes that (19a) may not be an argument against Ionin and Matushansky (2006) if we assume an atomizer semantics for measure terms. Even if we did, problems persist, though.

In (19a), the measure word *kilo* can only denote a measure expression by combining with a numeral. We surely would not wish to say that *hundred* restrictively modifies *kilo*, giving rise to a *subset-of-kilo* interpretation, not to mention that measure words like *kilo*, denoting a measuring dimension, are not predicates of the appropriate type. Also, Ionin and Matushansky's (2006) model incorrectly predicts *hundred kilos* (without a preceding numeral), and *three a hundred kilos* to be grammatical. After all *hundred kilos of flour*, being of type <e,t>, should successfully denote a property; and there is nothing to prevent the unattested *three a hundred kilos* given that the <<e,t>, <e,t>>-type *three* can successfully combine with the <e,t>-type *a hundred kilos*. Also, modifiers are in general optional while numerals must obligatorily occur in measure constructions. Finally, (19b) references to numbers as entities, rather than some modifiers, contra Ionin and Matushansky (2006).

In general, both the interpretation and the distribution of numerals are rather different from those of modifiers. We can therefore conclude that the numerals-as-modifiers analysis is untenable.

3.1.4 Numerals as numbers of type n

The last account of numerals, proposed in Krifka (1995), Landman (2004), Rothstein (2011; 2017), Scontras (2014), and Sağ (2019) takes a Platonistic view in treating them to be type *n* entities, referring exclusively to natural numbers. Scontras (2014) defines measurement as a function that locates entities on an interval along a dimension. The syntactic correlate of a measuring expression is a μP, whose head is filled by measure words. The interval, which can be

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(i) Masa-da kilo*(-lar-ca) un var. (Turkish)
table -LOC kilo-PL-DRV flour exist
'There is kilo(s) of flour on the table.'

If *kilo un* 'kilo flour' were to denote a set of atoms weighing a kilo each, why can it not function as a predicate without a preceding numeral or the following plural marker plus the derivational morpheme? It appears that an atomizer semantics may be appropriate for derived measure expressions like *kilo-lar-ca* 'kilo-PL-DRV', but not for those occurring with numerals. Note in the passing that Scontras' (2014) measuring-to-counting shift (by which measure terms function as countable containers referring to entities) is also unavailable in Turkish.

(ii) a. I bought two beautiful liters of wine.

(Scontras, 2014, p. 55)

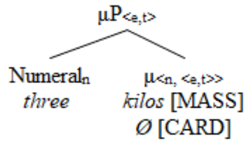
b. *İki güzel litre şarap al-dı-m.
two beautiful liter wine buy-PST-1SG
Int.: 'I bought two beautiful liters of wine.'

conceived of as a degree along the dimension, is specified by numerals. It thus comes as natural that measure expressions necessarily involve a numeral. Overall, μP denotes a property, as evidenced by the fact that they can be used attributively as well as predicatively.

(20) a. *three kilos* of apples
b. The apples are *three kilos*.

Measure expressions in Scontras (2014) are viewed as relations between numbers and individuals. One way of putting this is by saying that they are <n, <e,t>>-type, unsaturated objects which take a numeral as input and return a property as output, which is then attributed to an entity. Scontras (2014) goes on to propose that cardinality is also a measure function, introduced by the μ-head CARD. If the μ° is filled by *KILO*, we have a measurement along the mass dimension, and if it is filled by *CARD*, we have a measurement along the cardinality dimension. The structure of μP is given in (21).

(21) the μP



Note that this model, being the null hypothesis, is far superior to the other accounts of numerals. Denoting natural numbers, numerals can easily be used to calculate, to count, and to participate in measuring contexts where they complete the meaning of μ° by specifying the interval argument of the dimension. Above all, it is a welcome result that numerals are assigned a different semantic type than determiners, predicates, and modifiers, all of which have been demonstrated to pattern differently from numerals in distribution as well as interpretation. One further advantage of this model is that it has the potential to accommodate classifier languages, whose numerals must be accompanied by a classifier. All we need to assume is that classifiers spell out CARD, as proposed for Turkish classifiers in Sağ (2019) and Turgay (2020).

The account of numerals I develop in the upcoming sections is thus inspired by the view that all forms of numerals (simplex or complex, additive or multiplicative) ultimately denote type *n* entities.

3.2 The NumP

In this section, I elaborate on the internal constituency of numerals. I start with the null hypothesis that complex numerals like *three thousand five hundred and twenty-seven* are syntactic phrases. The phrasal status of complex numerals is evident in several respects. First, they involve coordinators like *and*. Second, different morphemes have been observed to appear in between parts of complex numerals. In Turkish, for instance, the distributive marker *–(ş)Ar* can be suffixed at the end of a complex numeral (22a) as well as after the first constituent (22b) (Lewis, 1967; Kornfilt, 1997; Göksel & Kerslake, 2005).

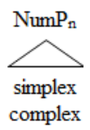
(22) a.	üç	yüz-*er*	lira	
	three	hundred-DIST	lira	
	‘three hundred liras each’			
b.	üç-*er*	yüz	lira	
	three-DIST	hundred	lira	
	‘three hundred liras each’			

If numerals were non-phrasal lexical atoms, it would be a theoretical challenge to explain how the distributive marker surfaces inside of them as in (22b).

Third, considering that numerals occur as arguments to μ heads, they must be phrases themselves, as no head can possibly take another head as an argument.

Having established that complex numerals are *n*-type constituents realized as phrases in syntax, by Uniformity Principle, simplex ones like *eight* must also be *n*-type syntactic phrases, given that both types of numerals perform the same function. We thus arrive at (23) for the syntax and semantics of numerals, simplex or complex.

(23) Structure of NumP



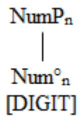
Note that this representation is agnostic about the internal constituency of numeral constructions, and in particular, how complex numerals are derived additively or multiplicatively. This is addressed in the next section.

3.3 Derivation of Simplex and Complex Numerals

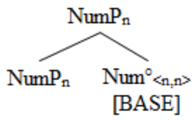
I assume with earlier works that numerals are built on two basic primitives: DIGITs like *one*, *two*, *three*, …, *nine*, and Ms (henceforth BASEs) like *ten*, *hundred*, *thousand*, *million*, etc.

DIGITs are *n*-type, saturated entities that project directly to NumP, whereas BASEs are <n,n>-type, unsaturated entities that must take an *n*-type entity as an argument.[5] Both DIGITs and BASEs fill in the Num° position. The former derives simplex numerals, and the latter derives one form of complex numerals.[6] The [NumP-BASE] sequence is interpreted multiplicatively. The numerals built on DIGITs and BASEs are as follows.

(24) a. Simplex numerals



b. Complex multiplicative numerals



The difference between a DIGIT and a BASE is that of transitivity, also observed in other aspects of grammar. Some verbs like *run*, for instance, are intransitive <e,t>-type objects, while others like *clean* are transitive objects of type <e, <e,t>>. The latter group can only “complete” their meaning after being “saturated” by the presence of a phrasal NP. Crucially, though, both are verbs.

---

[5] A reviewer asks how the system can capture the interpretation of pluralized bases like *yüzlerce* ‘hundreds’, if bases are type <n,n>. I do not have a straightforward answer at the moment, but some preliminary remarks are in order.

(i) a. (*beş) kilo-lar-ca elma
five kilo-PL-DRV apple
‘(*five) kilos of apples’

b. (*beş) yüz-ler-ce kilo elma
five hundred-PL-DRV kilo apple
‘(*five) hundred kilos of apples’

Observe in (i) that pluralization can apply to measure terms like *kilo* as well as to bases like *hundred* and that both of them reject numerals. In light of this, I propose, with special thanks to the reviewer, that the plural marker introduces an *n*-type object denoting an indeterminate high numeral (I will not take a position on the role of *-cA* here), which then saturates the *n* argument of the base/measure word. In that sense, *kilolarca elma* and *yüzlerce kilo elma* roughly mean “many kilos of apples” and “many hundreds of kilos of apples” respectively.

It should be noted however that pluralizing the classifier in (ii.a) or combining it with a pluralized base in (ii.b) lead to ungrammaticality in some speakers’ idiolect, contrary to what this proposal predicts.

(ii) a. #tane-ler-ce elma
CL-PL-DRV apple
Int.: ‘numerous apples’

b. #yüz-ler-ce tane elma
hundred-PL-DRV CL apple
Int.: ‘many hundreds of apples’

Under the assumption I made earlier with Scontras (2014) that both measure words and classifiers are μ-heads of type <n,<e,t>> (see (21) above), it is a mystery why pluralization can apply to the former but not to the latter. Given the limits of this paper, I leave the issue open here, hoping to address it in a future study.

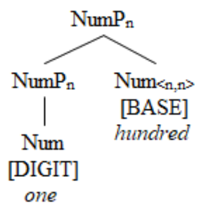
[6] See Rothstein (2017) for a similar analysis that takes DIGITs and BASEs to be of different types.

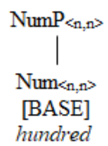
Similarly, both DIGITs and BASEs are numeral expressions, the only difference being that the latter is unsaturated. Ultimately, both yield an *n*-type NumP (24), and thus can appear in all positions reserved for numerals.

(25) a. $[_{\text{NumP}}$ three] kilos of apples (measuring)
a'. $[_{\text{NumP}}$ three hundred] kilos of apples
b. $[_{\text{NumP}}$ Three] plus $[_{\text{NumP}}$ seven] equals $[_{\text{NumP}}$ ten]. (calculation)
b' $[_{\text{NumP}}$ Three hundred] plus $[_{\text{NumP}}$ seven-ty] equals $[_{\text{NumP}}$ three hundred seventy].

Several notes are in order here. First, recall that numeral expressions *ten*, *hundred*, *thousand*, *million*, etc. are all BASEs, and that BASEs always require a NumP to be saturated. This means that, for a BASE to denote a numeral, an *n*-type NumP must always be present in its complement position. I therefore assume, in contrast to Hurford (1975; 1987; 2007) and Booij (2009), that a numeral expression like *hundred books* always involves a (sometimes null) NumP with the precise value of 1, and should therefore be represented as in (26a).[7]

(26) *hundred books*
a. one hundred books b. *hundred books





This is because, unlike the *n*-type root NumP in (26a), (27b) fails to denote a numeral for the simple reason that it is an unsaturated object of type <n,n>. This being the case, (26b) would not be able to participate in any context that numerals do. Further, we have morphological evidence that this is indeed so. In English, for instance, *hundred* as a numeral can also be stated as *one/a hundred*. In Po

[7] The question of why *bir* 'one' has to occur with *milyon* 'million' and higher bases is certainly an interesting one, an issue on which my analysis offers no explanation. Nevertheless, this paper is more interested in the derivation of complex numerals in a minimalist framework than why specific bases require (c.f. *(one) million), allow (c.f. (one) hundred), or even reject (c.f. *(*bir) yüz* 'one hundred') an overt 1, which is another research issue.

Tangle, on the other hand, the BASE for *10* is *kwi* (27a-a'). But in numerals up to 19, *gbọmọ* is used instead (27b-b').

(27)	a.	kwi	rap		a'.	kwi	pelau
		ten	two			ten	seven
		'twenty'				'seventy'	
	b.	gbọmọ	salai	rap	b'.	gbọmọ salai	pelau
		ten	and	two		ten and	seven
		'twelve'				'seventeen'	

(Amaechi, 2014, p. 41)

We can hypothesize that the NumP corresponding to *kwi dọk* 'ten one' is lexicalized as *gbọmọ* 'ten'. In a sense, *kwi* corresponds to English *-ty*, and *gbọmọ* to English *ten* (or *-teen*).

Second, whether the NumP that serves as a complement to the BASE occurs before or after it is governed by the directionality parameter. Apparently, Indo-European languages as well as Turkish are head-final in that respect whereas Po Tangle is head-initial.

(28)	a.	kwi	kunung	b.	won	lambuda	c.	lakikintham	padau
		ten	three		hundred	nine		thousand	four
		'thirty'			'nine hundred'			'four thousand'	

(Amaechi, 2014, pp. 41-42)

Third, although the BASE *ten* tends to fuse with the complement NumP in many languages, which sometimes makes the isolation of the morphemes impossible as in Turkish *yirmi* 'twenty', I still assume that they are to be represented as in (26a). It is a matter of Spellout whereby fusional morphemes like English *eleven* and Turkish *yirmi* 'twenty' win the competition at lexical insertion against isolable ones. Such fusional patterns are known to closely interact with frequency of use (Aronoff & Anshen, 2001; Bauer, 2004; Bybee, 2007; Fernández-Domínguez, 2010), and it should come as no surprise that we observe them with lower-valued numerals.

Finally, observe that the derivation here proceeds in accordance with the requirements of the lexical items, as dictated by Minimalist desiderata. In simplex numerals, a DIGIT necessarily projects to a phrase by Headedness Principle; while in complex numerals, a BASE requires its argument to be realized as early as possible, hence the obligatory presence of a NumP in its complement position (c.f. (24b)). It is only after this saturation that the BASE-headed Num° can close off with its maximal projection of NumP.

Having discussed the syntax of simplex and multiplicative interpreted complex numerals, let us now move onto the derivation of additively interpreted complex numerals. Recall that these numerals typically involve the (sometimes optional) presence of a coordinator, which prototypically happens to be *and* or one of its kin.

(29) a. three hundred (and) for-ty five (English)

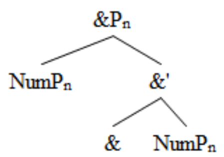
b. een-en-vijftig (Dutch)
one-and-fifty
'fifty-one'
(Booij, 2009, p. 9)

c. won puwad ka kwi padau salai payindi (Po Tangle)
hundred five and ten four and six
'five hundred and forty-six'
(Amaechi, 2014, p. 42)

d. otuz artukı tokuz (Old Turkic)
thirty and nine
'thirty-nine'
(Kaymaz, 2002, p. 750)

In accordance with the literature, I propose that the syntax of additive numerals involve coordination, represented syntactically as &P.[8]

(30) Complex additive numerals

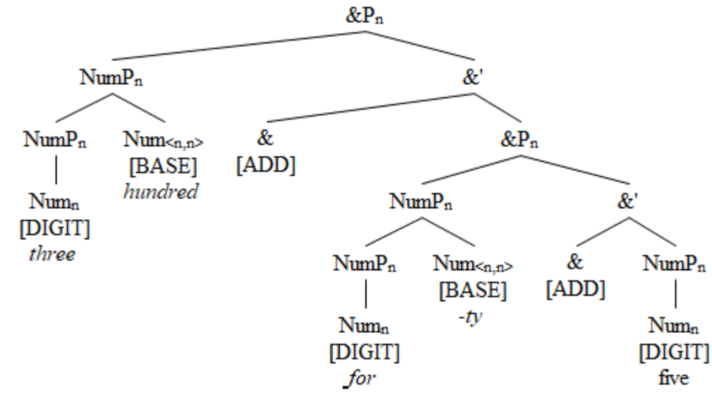


The &P, not being associated with a semantic type, inherits its type from its complements, and thus surfaces as *n*. The &° is typically filled by the additive coordinators but it can also be subtractive (see Section 4). Since &P selects for phrasal NumP arguments, it is not sensitive to whether its complements are maximal projections of DIGITs or BASEs.

Given (30), the derivation of a complex numeral like *three hundred (and) forty-five*, which involves both multiplication and addition, goes as in (31).

[8] See Tatsumi (2021) for a non-coordination analysis of additive numerals in Japanese and Chinese.

(31) *345*

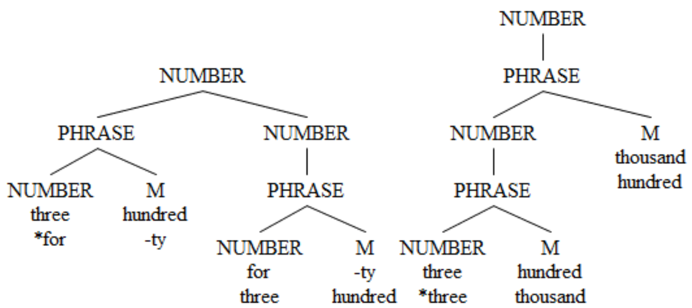


In the next section, we will reinterpret Hurford's PS in light of the model we developed so far, and discuss how it constrains the order of NumPs relative to one another.

3.4 Packaging Strategy Revised

Recall from Section 2.1 that Hurford (1975; 1987) proposed PS as a well-formedness condition on the output of the generative mechanism that produces numerals. It was argued that, by PS, higher-valued numerals appear structurally higher than lower-valued ones, i.e. they are "packed into the structure nearer the top of the phrase structure tree" (Hurford, 2007). This results in higher-valued NUMBERs, our NumPs, to appear before lower-valued ones when additively combined. The order is reversed in multiplicatively combined numerals. The relevant data is repeated below for convenience.

(32) a. Additive b. Multiplicative
340 *300,000*



In the additive (32a), the higher-valued *three hundred* appears structurally higher than the lower valued *forty*. Likewise, in the multiplicative (32b), the higher-valued *thousand* appears higher than the lower-valued *three hundred*. By PS, though, *three hundred* is ordered before *forty* in (32a), and *thousand* is ordered after *three hundred* in (32b).

Also, recall Hurford's argument that this sort of ordering is ensured by combining NUMBERs "top-down", starting with higher-valued constituents. We have seen, however, that top-down derivations induce a "look-ahead" problem, given the Minimalist assumption that derivations work bottom-up, starting with the most-deeply embedded constituent. To fix this problem, all we need to do is to restate PS in a way that is consistent with a bottom-up derivation. Further, instead of forcing the system to "find out" the higher-valued NumPs by calculating their values through multiplication, we can reduce the computational load on the system by merely requiring it to compare the BASEs in the relevant NumPs. Our revised Packaging Strategy then looks like (33).

(33) Revised Packaging Strategy (RPS)
When merging NumPs, start with the one involving the lowest possible BASE and work your way upwards.

By RPS, the system works bottom-up in (32a) by first taking the NumP *forty* involving the lowest BASE *10*, and then merging it with the next lowest NumP *hundred* involving the BASE *100*. As such, the ungrammatical [*forty* [*three hundred*]] (in which the derivation apparently starts with the NumP involving the higher BASE *100*) is ruled out as it violates RPS. Similarly, in (32b), the derivation starts by merging the NumP *three hundred* involving the relatively lower BASE *100* and then combining it with the next lowest BASE *1000*.

Let us now consider the derivation of *345*, for which the only acceptable Spellout is (34a).

(34) *345*

a.	three hundred forty-five	=	3x100+4x10+5
b.	*three hundred fifty-four	=	3x100+5x10+4
c.	*four hundred thirty-five	=	4x100+3x10+5
d.	*four hundred fifty-three	=	4x100+5x10+3
e.	*five hundred thirty-four	=	5x100+3x10+4
f.	*five hundred forty-three	=	5x100+4x10+3

Under the assumption that our numeration looks like {3, 4, 5, 10, 100}, what ensures that the DIGITs *3* and *4* combine with the BASEs *100* and *10* respectively? In other words, what prevents the ungrammatical derivations in (34b-f)? There are essentially two ways to avoid these undesired derivations. One is by assuming that the BASE Num°s *100* and *10* only select the NumPs involving the DIGITs *3* and *4* respectively. The other is by assuming that the numeration is not an unordered list, but rather structured into subarrays (Chomsky, 2000; 2001). In this view, the derivation of (34a) proceeds as follows.

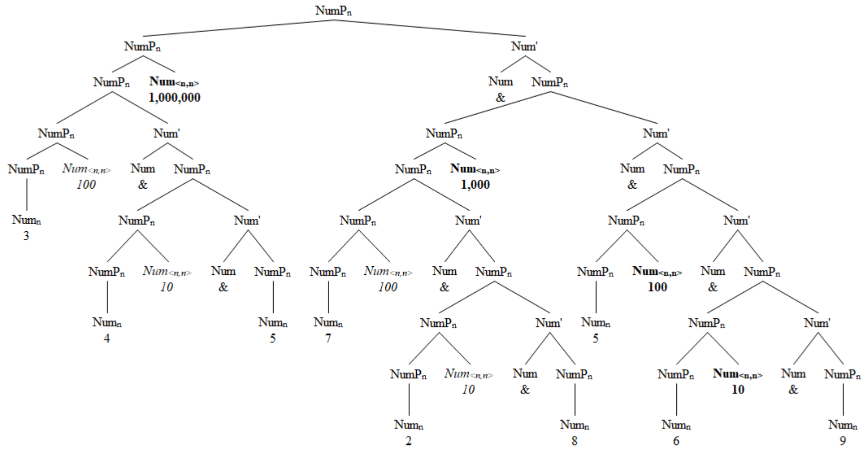
(35) The derivation of *345*
   a. the numeration
      {**{3, 100}**, **{4, 10}**, 5}
   b. first cycle
      {3x100, 4x10, 5}
   c. second cycle
      3x100+4x10+5
   d. Spellout
      three hundred forty-five

The system starts with the numeration in (35a). It first activates the subarrays in bold and merges them in parallel, giving us the partially derived numeration in (35b). Since these subarrays involve BASEs, their merger is necessarily multiplicative. It is only after all the subarrays are exhausted that the system moves on to the merger in (35c). Given that we have no BASEs left, the result will always be additive. The derivation is eventually spelled out as (35d), which is identical to the only grammatical derivation in (34a).

With the assumption that numeration is structured into subarrays, we can also capture the derivation of the complex numeral in (36).

(36) *345,728,569*
   a. the numeration
      {{{{3, *100*}, {4, *10*}, 5},**1000000**}, {{{7, *100*}, {2, *10*}, 8}, **1000**}, {5, **100**}, {6, **10**}, 9}
   b. first cycle
      {{{3x100, 4x10, 5},**1000000**}, {{7x100, 2x10, 8}, **1000**}, {5, **100**}, {6, **10**}, 9}
   c. second cycle
      {{3x100+4x10+5, **1000000**}, {7x100+2x10+8, **1000**}, {5, **100**}, {6, **10**}, 9}
   d. third cycle
      {345x**1000000**, 728x**1000**, 5x**100**, 6x**10**, 9}
   e. fourth cycle
      345x1000000+728x1000+5x100+6x10+9
   f. Spell out
      three hundred forty-five million seven hundred twenty-eight thousand five hundred sixty-nine

g. Structural representation



Note crucially that these italicized BASEs are not at the same level with the bold BASEs of the numeration. Accordingly, the system first activates the most deeply embedded subarrays involving the italicized BASEs as in (36a) and merges them in the first cycle. The second cycle involves the additive coordination of the output of the first cycle. In accordance with RPS in (33), this additive merger proceeds bottom-up, starting with the NumPs involving the lowest BASEs (the *5* and the *8* involving no BASEs) and combining them with the next lowest ones (the *10*s). This is clearly reflected in the structural representation in (36g). The third cycle further reduces the numeration to same-level items involving only the bold BASEs. The resulting NumPs are then coordinated in the fourth cycle, in accordance with RPS, and then spelled out as in (36f). Crucially, both additive and multiplicative mergers work bottom-up.

One further point to note at this conjecture is that the italicized BASEs *100* and *10* in the complement position of the bold BASEs *1,000,000* and *1,000* become invisible by the time the bold BASEs are brought into the picture. If this were not the case, these italicized BASEs *100* and *10* would necessarily occur after the bold *1,000,000* and *1,000*, leading to ungrammatical derivations. Such undesired outcomes are prevented in principle by the assumption that BASEs cannot "see" the internal structure of their complement NumPs.

We have seen that the revised, Minimalist, bottom-up version of Packaging Strategy captures the same set of data that Hurford (1975; 1987; 2007) intended to.

3.5 More on BASEs

We have so far been merely assuming the existence of BASEs like *10*, *100*, *1000* as lexical primitives upon which complex numerals are built. Is there a principled explanation as to why so many languages have these as their basic BASEs? Could it have been otherwise? In particular, why does no known language use the combination of two or three *10*s as possible BASEs for *100* and *1000* respectively? In such a fictitious English*, the numerals in (37) would be spelled out as follows.

(37) a. *345*
*three [ten ten] forty-five
= 3x10x10+4x10+5
b. *3,456*
*three [ten ten ten] four [ten ten] fifty-six
= 3x10x10x10+4x10x10+5x10+6

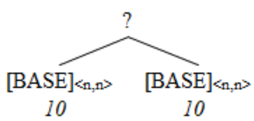
Why do we never observe such a pattern?

In Hurford (2007), the ungrammaticality of (37a-b) follows from the principle *Minimize entities you are dealing with* (c.f. (8b)). The larger metaphoric "basket" of *100* is taken to be a minimized version of two *10*s multiplied. In Booij (2009), the unavailability of such patterns is explained as a condition of economy, whereby the structurally simpler *100* precludes the complex *10x10*. Hurford's (2007) principle can also be restated in terms of similar economy considerations.

Note that this view of derivational simplicity comes at the expense of increasing lexical storage. More importantly, the use of *100* becomes more economical than *10x10* only "after" the former is created and stored in the lexicon. We, however, are concerned with the stage "before" *100* becomes available as an independent BASE competing with *10x10*. At this stage, the issue of economy should not come up at all. Given this, why does English* not express the relevant BASE as *10x10* but opts for the creation and subsequent storage of the alternative *100*? In what follows, I provide an explanation that naturally follows from the model presented so far.

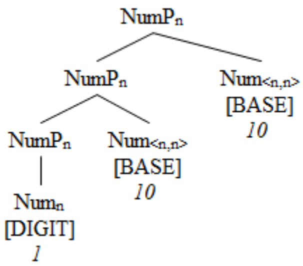
Recall that all BASEs are unsaturated entities of type <n,n>. This predicts the combination of any two BASEs to result in ungrammaticality due to type clash.

(38) Type clash with juxtaposed BASEs



This makes the simple juxtaposition of two BASEs impossible. To express *100*, our hypothetical English* can nevertheless still use two *10*s, one of them taking a NumP argument before composing with the other, as in (39).

(39) Alternative derivation with same-BASEs



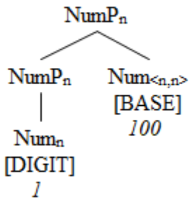
Observe however, that what we end up with here is not a BASE of type-<n,n>, but a full-fledged NumP of type-*n*, namely the *1x10* complement of the highest *10*. Thus, (39) is essentially the outcome of a syntactic process, not a lexical one. As such, it cannot be considered a lexical BASE.[9]

In brief, what I have demonstrated is that, when we arrive at *100*, we have an apparent choice between using the BASE *10* twice and introducing a new BASE equal in value, namely *100*. Since we do not have *100* in the lexicon yet, the system cannot compare the two alternative derivations in terms of economy. Therefore, the unavailability of the first option must be accounted for by means other than economy. This, I believe, is type clash shown in (38). This state of affairs leaves us with the second option only: creating a new BASE. Once we have two BASEs within reach, the derivation in (39) involving two instances of *10* competes with the alternative derivation in (40) involving *100*.

---

[9] It turns out that BASEs can be idiosyncratic. *Twenty* can function as a productive BASE in French (Hurford, 2007), Mixtec, and Yoruba (Ionin & Matushansky, 2019), and multiples of *forty* can do so in Hawaiian (Hurford, 2007). If it is possible to take an *n*-type NumP like *twenty* and type-shift it into an <n,n>-type BASE, why is a similar process not possible in (39), a reviewer asks. As Hurford (2007) himself notes, these BASEs are highly idiosyncratic, with a rather narrow distribution both inter- and intra-linguistically. More importantly, the model developed here forces us to conclude that no NumP is type-shifted into a BASE, but rather that *twenty* exists in these languages as a lexical BASE, on a par with multiples of *ten*. In Hawaiian, for instance, 120 is expressed as *ekolu kanaha* ‘3x40’, demonstrating clearly that *kanaha* ‘40’ as a lexical BASE involves no DIGIT, making it quite unlike the derived NumPs in (39) or (40).

(40) Derivation of *100*



Since (40) contains fewer derivational steps than the same-value (39), it is preferred as the Spellout for *100*. In simpler terms, (38) is ruled out in principle, whereas (39) loses to (40) for reasons of economy.

The validity of this argument receives further support from diachronic studies. Old Turkic, for instance, has a separate BASE for *10000*, namely *tümen* (Kaymaz, 2002; Erdal, 2004).

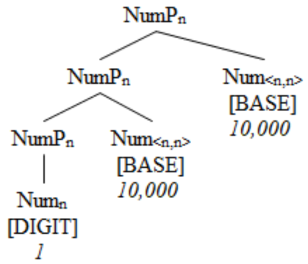
(41) a.	beş	*tümen*	b. min	*tümen*	c. *tümen*	*tümen*
	five	10,000	thousand	10,000	10,000	10,000
	'50,000'		'10,000,000'		'100,000,000'	

(Kaymaz, 2002)

(41a-b) are standardly derived as in (40), with the complement of *tümen* '10,000' being a full-fledged NumP of type *n*. Of particular relevance is (41c) where the BASE *tümen* '10,000' is repeated. The only way to save this construction from an inevitable type clash (c.f. (38)) is by assuming the syntax in (39). Recall though that (39) lost to (40) for reasons of economy. How about the grammatical (41c) involving the same derivation? Should it not be uneconomical like (39)? Not necessarily. Economy considerations do not come into picture in this case because old Turkic did not have a higher BASE to spell out *million*[10]. As such, (42) has no alternative derivation to be judged against in terms of derivational economy.

[10] Recall that, for reasons of type clash which disfavors *10x10*, all known languages have chosen to create *100* as an alternative BASE. A natural question at this point is why Old Turkic did not choose to create a higher BASE instead of using the highest BASE *tümen* '10,000' twice? Apparently, the introduction of new BASEs closely interacts with frequency: The more frequent a numeral is used, the more obligatory it becomes to introduce new BASEs. Thus, *milyon* 'million' was only introduced and replaced *tümen tümen* 'ten thousand ten thousand' when the high numerals became more frequent in use.

(42) Old Turkic *100,000,000*



Having no rival, (42) surfaces as the only derivation for *100,000,000* in old Turkic. This is reminiscent of physicist uttering expressions like *ten trillions, trillions, trillions, trillions of stars in the universe*, for lack of higher lexicalized bases to use.

In sum, in contrast to Hurford (1975; 1987; 2007) and Booij (2009), repeated BASEs are a priori ruled out in the model advocated here due to type requirements. Economy still plays a significant role in comparing alternative derivations and selecting the optimal one, but it only becomes relevant after different BASEs are introduced (c.f. (39) vs (40)), and in the presence of competing derivations (c.f. (39) vs (42)).

4 Other Forms of Interpretation

We have so far been discussing numerals involving the arithmetic operations of addition and multiplication. Do languages also use subtraction and division? It turns out that they do, though to a lesser extent. In this section, I briefly review some data and tentatively propose a possible extension of the model I have developed.

4.1 Subtractive Numerals

Dialects of old Turkic languages, as well as some still spoken today, use subtraction as an interpretive mechanism in the derivation of complex numerals. There are two basic modes of subtractive numeral formation. One is very similar to addition in that the DIGIT-based NumP is subtracted from (instead of being added to) the BASE-based one, as in (43a), a pattern also observed in Yoruba (43a'). The other, observed in Old Turkic, involves both addition to and subtraction from the BASE-based NumP, as in (43b-b').

(43) a. bir *kem* elli[11] = 5x10-1 (Turkmen)
one minus fifty
'forty-nine'

a'. merin-*din-ni*-aadota = 6x10-4 (Yoruba)
four-subtract-from-sixty
'fifty-six'

(Amaechi, 2014, p. 43)

b. tört otuz = 3x10-1x10+4 (Old Turkic)
four thirty
'twenty-four

b'. iki yüz üç tokuz on = 2x100+9x10-1x10+3
two hundred three nine ten
'two hundred eighty-three'

(Kaymaz, 2002, pp. 751-752)

In (43a-a'), the numerals *forty-nine* and *fifty-six* are derived by subtracting *1* and *4* from *50* and *60* respectively.

As for the derivation of Old Turkic (43b-b'), I assume, with Ionin and Matushansky (2006) (i) that numerals have a uniform interpretation inter- and intra-linguistically, i.e. that *tört* 'four' and *otuz* 'thirty' in (43b) retain their usual interpretations of *4* and *30* respectively, and (ii) that the interpretation of all complex numerals are compositionally derived from their constituent parts. With these assumptions, we are in fact forced to the conclusion that, to obtain the desired interpretation, a silent numeral with the precise value *10* is subtracted from the *10*-based NumP in the relevant examples. I therefore propose tentatively that in (43b), *twenty-four* is derived by first invisibly subtracting *10* from *30* and then adding *4*, invisibly in the sense that the subtracted *10* receives no phonological exponence. A similar derivation is involved in (43b'): First, *10* is invisibly subtracted from *90*, the result is then added to *3*, and finally to *200*[12]. Note that any other derivation will necessarily involve added stipulations that might go as far as to abandon compositionality or uniform semantics for numerals.

Considering (43a-b'), it appears that the ordering of NumPs relative to one another, and by extension the order of interpretation, is the exact opposite of

[11] Kaymaz (2002, p. 760) notes that these subtractive numerals have a restricted distribution, used primarily to tell the age of a person.

[12] This pattern is described in Turkish grammars as "the next higher decimal system" (Kaymaz, 2002), squaring with our proposal that some sort of *10*-subtraction or its equivalent must be at work.

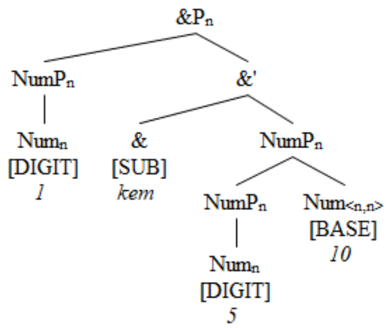
additively derived numerals[13], in violation of Hurford's (1975; 1987) PS and our RPS. (43b') establishes, however, that appearances are misleading. The reversed order of coordinated NumPs is only allowed when combining DIGITs *1-9* with numerals based on the BASE *10*. NumPs involving higher BASEs still appear structurally higher and thus occur to the left, as in (43b'). Such an apparent violation of (R)PS is also observed in the additive numerals of German.

(44) *345*
drei hundert *fünf und vierzig* = 3x100+4x10+5 (German)
three hundred five and forty
'three hundred forty-five'

To accommodate this set of data, we need to lax RPS so as to allow additively and subtractively combined NumPs involving the BASE *10* to, language-specifically, occur structurally lower than the DIGIT-based ones. The special status of *10*-based numerals is also evident in the fact that they tend to fuse with surrounding DIGITs, as is the case with English *eleven*.

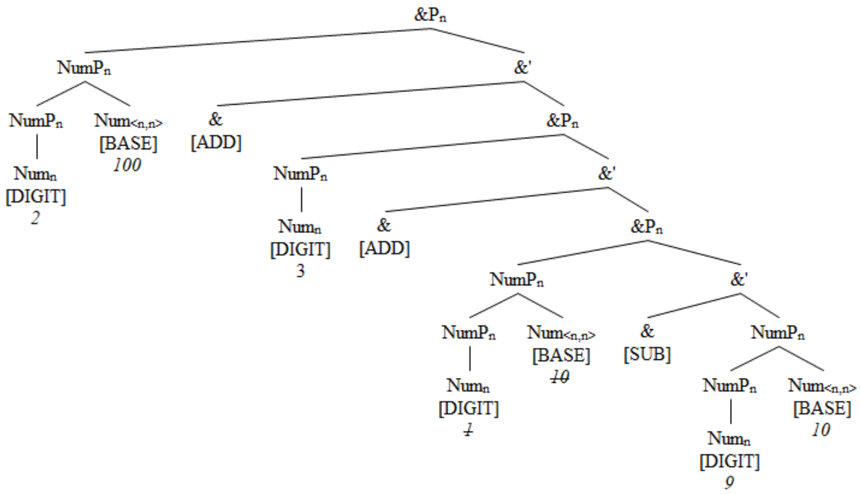
This being the case, I tentatively propose that (43a) and (43b') are derived as follows.

(45) a. *bir kem elli* 'one minus fifty' (Turkmen for *49*)



[13] Note that the reversed order of (43a), namely *elli kem bir* 'fifty minus one', is not a numeral but a mathematical formula. Likewise, the reversed version of (43b), i.e. *otuz tört*, only receives an additive interpretation meaning *thirty-four*.

b. *iki yüz üç tokuz on* 'two hundred three nine ten' (Old Turkic for *283*)



This is not the end of the story. As can be observed in (45b), the NumP valued *10* that is subtracted from *30* receives no phonological exponence. The exact workings of these numerals are beyond the scope of this paper. I hope that the preliminary hypotheses explored here provide a first step into a detailed analysis of these numerals.

4.2 Divisive Numerals

An even more complicated picture emerges with the formation and interpretation of divisive numerals, the prototypical examples of which are fractions. Consider the following data.

(46) a. üç *bölü* dört (Turkish)
three over four
'three over four'
b. dört-*te* üç
four-LOC three
'three fourths'
c. üç *virgül* dört
three comma four
'three point four'
d. iki *tam* üç bölü dört
two whole three over four
'two and three fourths'

Despite expressing the same fractional value, (46a) is the mirror image of (46b) in terms of the relative position of the numerals. (46c-d) involve other forms of expressing fractions.

Expanding on Hurford (1975; 1987), Koşaner (2016) posits the existence of a further Fraction Phrase to capture the distribution of fractions in Turkish.

(47) Expanded Phrase Structure Rules

a. $$\text{NUMBER} \rightarrow \begin{Bmatrix} \text{DIGIT} \\ \text{FRACTION PHRASE} \\ \text{PHRASE} \end{Bmatrix}$$

b. $$\text{FRACTION PHRASE} \rightarrow \text{NUMBER FRACTION}$$

Koşaner (2016) goes on to provide a syntactic model of how fractional numerals are derived. Since, however, he employs phrase structure rules, with no discussion of how the parts of fractions are semantically composed to yield the resulting interpretation, I will not go into the details of his analysis here. I personally feel that fractions should not be treated on a par with other numerals, and fully accept that their formation and interpretation involve mechanisms too intricate to be easily subsumed under the model proposed in this study. I hope to address the derivation of these constructions in future work.

5 Conclusion

This study addressed the internal syntax and emerging semantics of numerals. After reviewing Hurford's (1975; 1987; 2007) and Booij's (2009) accounts in Section 2 and highlighting areas of improvement, I then moved in Section 3 onto a Minimalist account of numerals. It was first demonstrated in Section 3.1 that the occurrence of numerals in a wide range of contexts can best be accounted for by assuming Krifka (1995), Rothstein (2011), and Scontras' (2014) proposal that they are type-*n* entities denoting natural numbers, and that they occur in counting contexts only as an argument of the counting function CARD, itself a possible value of μ°. It was then established in Section 3.2 that NumP, the syntactic maximal category of numerals, can be headed by two items: saturated intransitive DIGITs of type *n*, and unsaturated transitive BASEs of type <n,n>. The latter form was shown not to denote a numeral without taking another NumP as a complement. It was illustrated in Section 3.3 that this head-complement relation is interpreted multiplicatively, whereas coordinated forms are interpreted additively. I then revised Hurford's (1975; 1987; 2007) Packaging Strategy in Section 3.4 to allow bottom-up derivation, in keeping with Minimalist requirements. Section 3.5 elaborated on the mere existence of BASEs, addressing the question of why so many languages have a number of BASEs like *100*, *1,000*, instead of simply combining *10*s multiple times. It was demonstrated in particular that a BASE cannot be combined with itself (or with another BASE) for the simple reason that, being of type <n,n>, their composition would not yield an interpretable linguistic object. It was further demonstrated that economy still plays a significant role in selecting the most "optimal" derivation. Two derivations are compared for economy only if one involves fewer BASEs. In case the language lacks lexical resources to reduce the number of BASEs, repetition of the same BASEs was shown to be possible. In Section 4, comments were provided with respect to a possible extension of this model into the syntax and semantics of subtractive and divisive numerals. Based on language-specific constraints, I also proposed to relax the Packaging Strategy to allow the combination of NumPs involving the BASE *10* with the ones involving simple DIGITs. With that in place, the status of subtractive numerals was shown to follow naturally from the model proposed in this study.

There is still room for research before we arrive at an overarching theory of numerals that captures their syntactic structure and semantic interpretation. In particular, the morphology, syntax, and semantics of divisively interpreted fractions defy common sense. It is yet to be seen what sort of entities they are and how they can be integrated into the emerging picture of numerals.

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